Chapter 2
Solow's Neoclassical Growth Model

2.1 Introduction

- The economy will more toward a stable **steady – state equilibrium**.

- In the steady – state equilibrium, there can be permanent economic growth only if there is technological progress.

- When the economy transitions from one steady state to another, medium – term growth in per capita output can occur.

- An economy will transition toward a higher steady state if there is an increase in its rate of saving or a decrease in its rate of population growth.

- An economy will experience higher permanent economic growth if there is an increase in its rate of labor – enhancing technological progress.

- The Solow model predicts only conditional convergence. Only if countries have the same saving rates, the same production functions, the same rates of technological progress, the same depreciation rates, and the same population growth rates can we be sure that their economies will converge to the same steady – state level of per capita output.

2.2 Assumptions

- There is only one commodity, output as a whole.

- Four variables include: output (**Y**), capital (**K**), labour (**L**) and "knowledge" or the "effectiveness of labor" (**A**).

- Production function: $\mathbf{Y(t) = F\left(K(t) , A(t)\, L(t) \right)}$ .... **(1)**

  where t denotes time.

- Features of the production function:

i) Output changes over time only if the inputs into production change.

ii) **AL** is referred to as effective labor, and technological progress that enters in this fashion is known as labor – augmenting.

iii) The ratio of capital to output ($\frac{K}{Y}$) being constant.

- Production function has **constant returns to scale** in both capital and effective labour.

i.e. **F (c K , c AL) = c F (K , AL)** for all **c** $\geqq$ **0** .... **(2)**

where **c** denotes any positive constant.

- Convert production function into per – worker form by setting **c** $= \dfrac{1}{AL}$ in **(2)**:

$$F \left(\frac{K}{AL} , 1 \right) = \frac{1}{AL} \ F \ (\ K , AL \ ) \ \text{......................} \ (3)$$

where $\dfrac{K}{AL}$ = amount of capital per unit of effective labor

$$\frac{F (K , AL)}{AL} = \frac{Y}{AL} = \text{output per unit of effective labor}$$

- Rewrite (3): **y = f(k)** .... **(4)**

where **k** $= \dfrac{K}{AL}$, **y** $= \dfrac{Y}{AL}$, **f(k) = F (k , 1)**

- Assumptions underlying the per – worker form production function:

a. **f '(k) > 0** for all **k** $\Rightarrow$ **MP$_k$ > 0**

b. **f ''(k) < 0** for all **k** $\Rightarrow$ **MP$_k$** is positive but declines as capital per unit of effective labour rises.

c. $\lim\limits_{k \to \infty} f'(k) = 0 \Rightarrow$ when $k \to \infty$, $MP_k \to 0$

d. $\lim\limits_{k \to 0} f'(k) = \infty \Rightarrow$ when $k \to 0$, $MP_k \to \infty$

e. $f(0) = 0 \Rightarrow$ No output can be produced without capital.

f. $f(\infty) = \infty \Rightarrow$ Indefinitely high level of output per worker is associated with indefinitely large ratio of **k**.

- Cobb – Douglas production function:

$$F (K , AL) = K^{\alpha}(AL)^{1-\alpha} \text{ where } 0 < \alpha < 1 \quad \text{(5)}$$

- Characteristics of Cobb – Douglas production function:

  i It exhibits constant returns to scale as the Solow model requires.

  $$F (c K , c AL) = (c K)^{\alpha} (cAL)^{1-\alpha} = c^{\alpha} K^{\alpha} c^{1-\alpha} (AL)^{1-\alpha}$$

  $$= c F (K , AL) \quad \text{(6)}$$

  ii It can be conveniently written in per – worker terms.

  $$F (K , AL) = K^{\alpha} (AL)^{1-\alpha} = K^{\alpha} \left[\dfrac{AL}{AL^{\alpha}} \right]$$

  $$f(k) \equiv F (\dfrac{K}{AL}, 1) = K^{\alpha} \left[\dfrac{AL}{AL^{\alpha}} \dfrac{1}{AL} \right] = k^{\alpha} \quad \text{(7)}$$

- Assume that the initial levels of capital, labour and knowledge are given.

- Labour and knowledge grow at constant rates:

$$\dot{L}(t) = n\, L(t) \quad \text{(8)}$$

and
$$\dot{A}(t) = g\, A(t) \quad\text{..} \quad (9)$$

where **n** and **g** are exogenous and positive parameters.

• A simple proportional saving function:

$$S = s\, Y \quad \text{where } 0 < s < 1 \quad\text{...........................} \quad (10)$$

• Investment is simply the rate of increase of the capital stock of the composite commodity:
$$\dot{K} = I \quad\text{..} \quad (11)$$

• Given that investment is identically equal to saving:

$$\dot{K} = S = s\, Y \quad\text{...} \quad (12)$$

• Now assume existing capital depreciates at constant rate **s > 0**:

$$\dot{K} = s\, Y(t) - \delta\, K(t) \quad\text{...} \quad (13)$$

2.3 The dynamics of the model

2.3.1 The dynamics of **k**

• Since $\mathbf{k} = \dfrac{\mathbf{K}}{\mathbf{AL}}$, thus:
$$\dot{k}(t) = \frac{\dot{k}(AL) - K(A\dot{L} + L\dot{A})}{(AL)^2}$$

$$\dot{k}(t) = \frac{\dot{K}(t)}{A(t)\,L(t)} - \frac{K(t)}{\left[A(t)\,L(t)\right]^2}\left[A(t)\,\dot{L}(t) + L(t)\,\dot{A}(t)\right]$$

$$= \frac{\dot{K}(t)}{A(t)\,L(t)} - \frac{K(t)}{A(t)\,L(t)}\left(\frac{\dot{L}(t)}{L(t)}\right) - \frac{K(t)}{A(t)\,L(t)}\left(\frac{\dot{A}(t)}{A(t)}\right) \quad\text{---------------}\quad (14)$$

• Subs. **(8) (9) (13)** into **(14)**:

$$\dot{k}(t) = \frac{s\,Y(t) - \delta\,K(t)}{A(t)\,L(t)} - k(t)\,n - k(t)\,g$$

$$= s\,\frac{Y(t)}{A(t)\,L(t)} - \delta\,k(t) - n\,k(t) - g\,k(t) \quad\text{\dotfill}\quad (15)$$

- Using the fact that $\dfrac{Y}{AL} = y = f(k)$:

$$\dot{k}(t) = s\,f(k(t)) - (n + g + \delta)\,k(t) \quad\text{\dotfill}\quad (16)$$

- **(16)** is the fundamental equation of the Solow model. It states that the rate of change of the capital stock per unit of effective labor is the difference between actual investment $s\,f(k(t))$ and breakeven investment $(n+g+\delta)\,k(t)$.
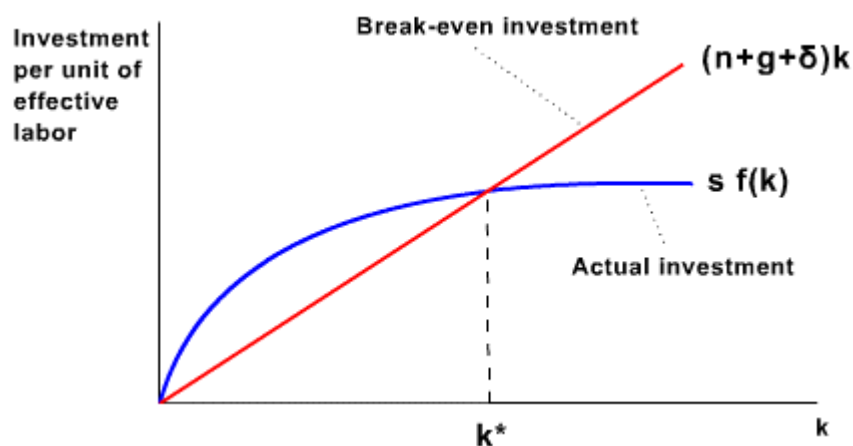
- At **k=0**, **f ' (k)** is large, and thus the **s f(k)** line is steeper than the $(n+g+\delta)\,k(t)$ line. Thus, for small values of **k**, actual investment is larger than breakeven investment.

- As **k** becomes large, **f ' (k)** falls towards zero. At some point, the slope of the actual investment falls below the slope of the breakeven investment line. With the **s f(k)** line flatter than the $(n+g+\delta)\,k(t)$ line, the two unit cross **k*** where actual investment and breakeven investment are also equal.
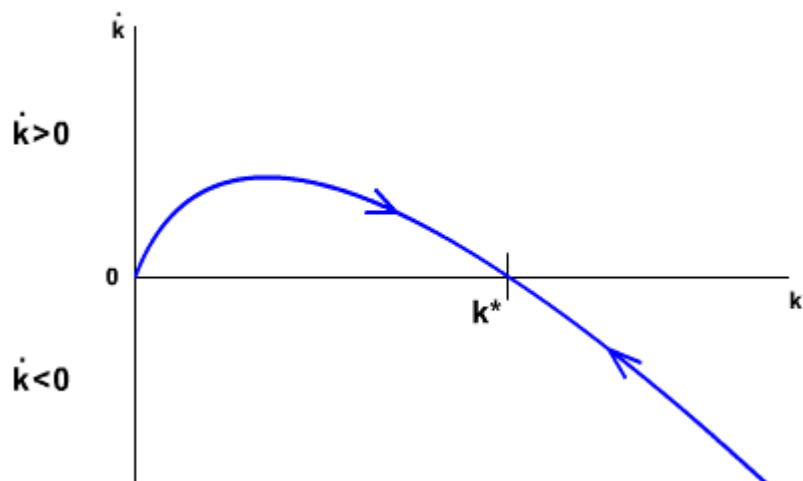
- When **k < k***, actual investment **>** breakeven investment, and so $\dot{k}$ is positive $\Rightarrow$ **k** is rising.

- When **k > k***, actual investment **<** breakeven investment, and so $\dot{k}$ is negative $\Rightarrow$ **k** is falling.

- When **k = k***, actual investment **=** breakeven investment, and so $\dot{k}$ is zero $\Rightarrow$ **k** is constant.

- Regardless of where **k** starts, it converges to **k***.

2.3.2   The balanced growth path

- The Solow model implies that, whatever the initial values of all the variables, the economy moves steadily towards a balanced growth path.

- A balanced growth refers to a situation where each variable of the model is growing at a constant rate.

- Form (16) , with $\dot{k} = 0$:

$$s\,f\big(k(t)\big) - (n + g + \delta)\,k(t) = 0$$

$$s\,f\big(k(t)\big) = (n + g + \delta)\,k(t) \dotfill (17)$$

- **(17)** shows that in steady state, savings per worker exactly match the steady – state

quantity of capital stock **k*** required to keep the growing labor force equipped. In steady state, the capital – labor ratio **k*** is constant.

2.4 The rate of saving and the steady state

- The long – run balanced rate of growth of a neoclassical economy is the constant exogenous rate of growth of both the labor force, **n**, and knowledge, **g**, and is entirely independent of the proportion of income saved.

2.4.1 The impact on output

- An increase in the propensity to save, **s**, shifts the actual investment line upward, and so **k*** rises.

- Initially at **k*old**, the income in the propensity to save raises savings per worker from A to D because at this level actual investment exceeds breakeven investment. This means more resources are being devoted to investment than are needed to hold **k** constant and hence, $\dot{k}$ is positive. An increase in capital is produced from D to E.

- The extra output creates, via the multiplier effect, additional income and expands savings per worker further from E to F. the extra saving in turn produces an increase in capital from F to G. these movements continue until a new equilibrium position **k*** is reached at B at which point **k** remains constant.

- A permanent increase in the saving rate produces a temporary increase in the growth rate of output per worker.
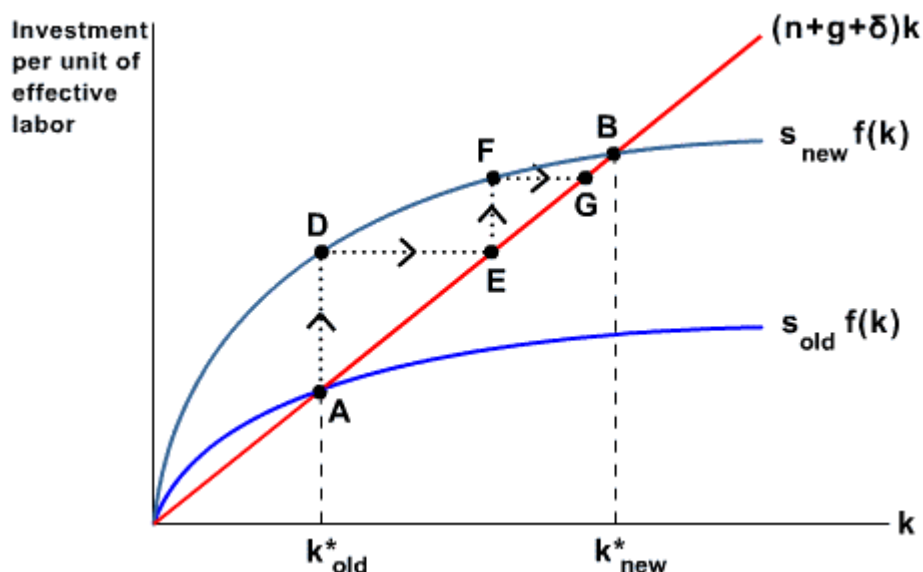
2.4.2 The impact on consumption

- Initially, consumption per unit of effective labor jumps downward. Then consumption rises gradually as **k** rises and **s** remains at its higher level.

- But whether consumption eventually exceeds its level before the rise in **s** is not clear. In the long run, it depends on whether **f '(k*)**, the marginal product of capital, is more or less than $n+g+\delta$.

- Let **c*** denotes consumption per unit of effective labor on the balanced growth path.

- The steady – state level of per capita consumption:

$$c^* = (1-s)\,f(k^*) = f(k^*) - s\,f(k^*)$$

- On the balanced growth path, $s\,f(k^*) = (n+g+\delta)\,k^*$:

$$c^* = f(k^*) - (n+g+\delta)\,k^* \quad\text{---(18)}$$

where $k^* = k^*(s,n,g,\delta)$

$$\frac{\partial c^*}{\partial s} = \left[f'(k^*(s,n,g,\delta))\right] - (n+g+\delta)\frac{\partial k^*(s,n,g,\delta)}{\partial s} \quad\text{-------------------(19)}$$

- When **f '(k*)** $< n+g+\delta$, the additional output from the increased capital is not

enough to maintain the capital stock at its higher level. Thus, an increase in the saving rate lowers consumption even when the economy has reached the new balanced growth path. In words, consumption must fall to maintain the higher capital stock.
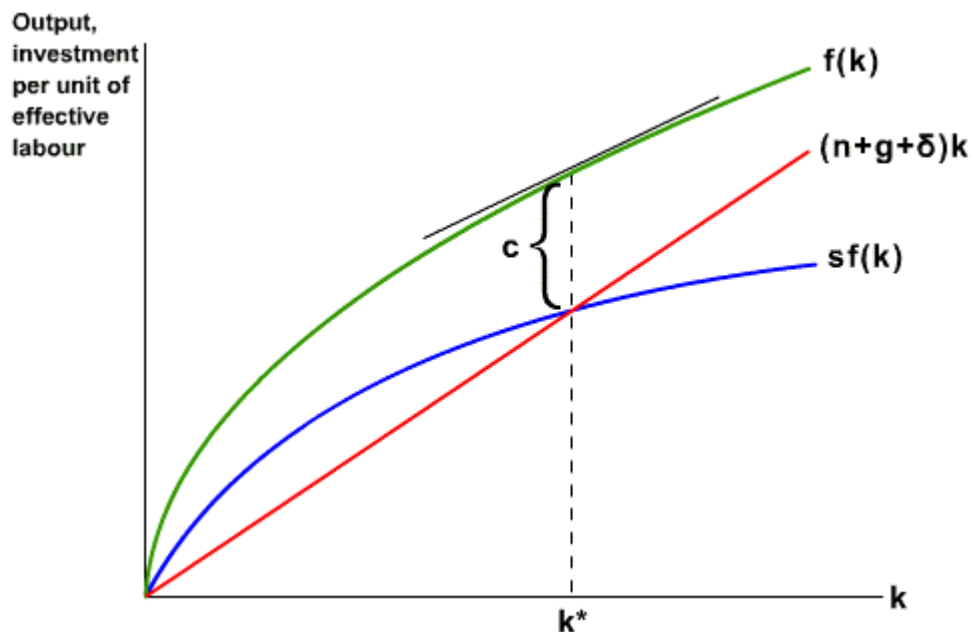
- When $f'(k^*) > n+g+\delta$, there is more than enough additional output to maintain **k** at its higher level. Thus, an increase in the saving rate raises consumption in the long run.

- When $f'(k) = n+g+\delta$, a marginal change in **s** has no effect on consumption in the long run, and consumption is at its maximum possible level among balanced growth paths. In term of human welfare, the steady state with the higher level of consumption per worker is known as the Golden Rule level of per worker output worker output and capital.
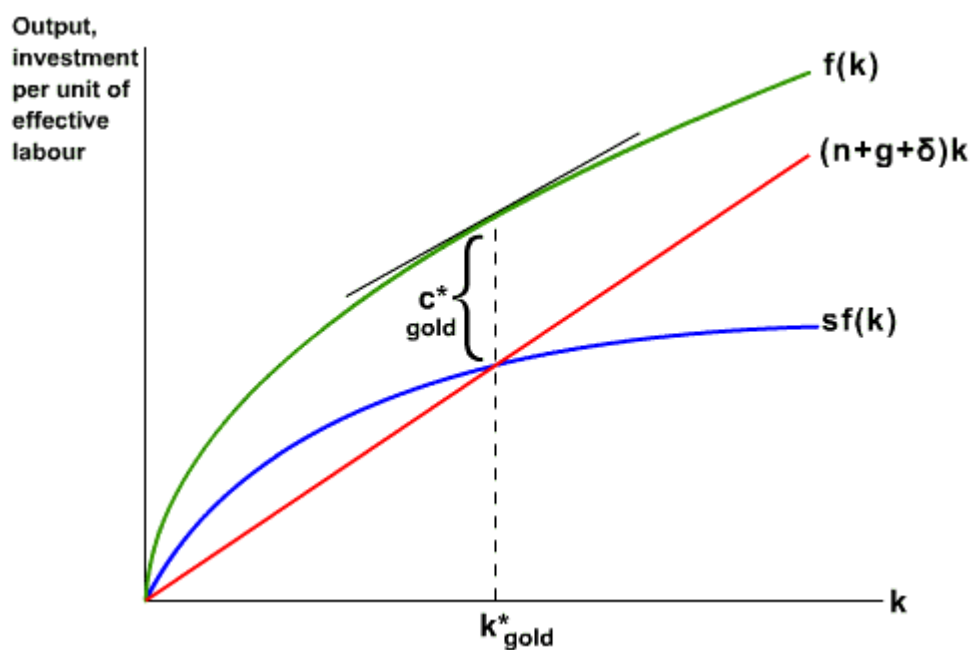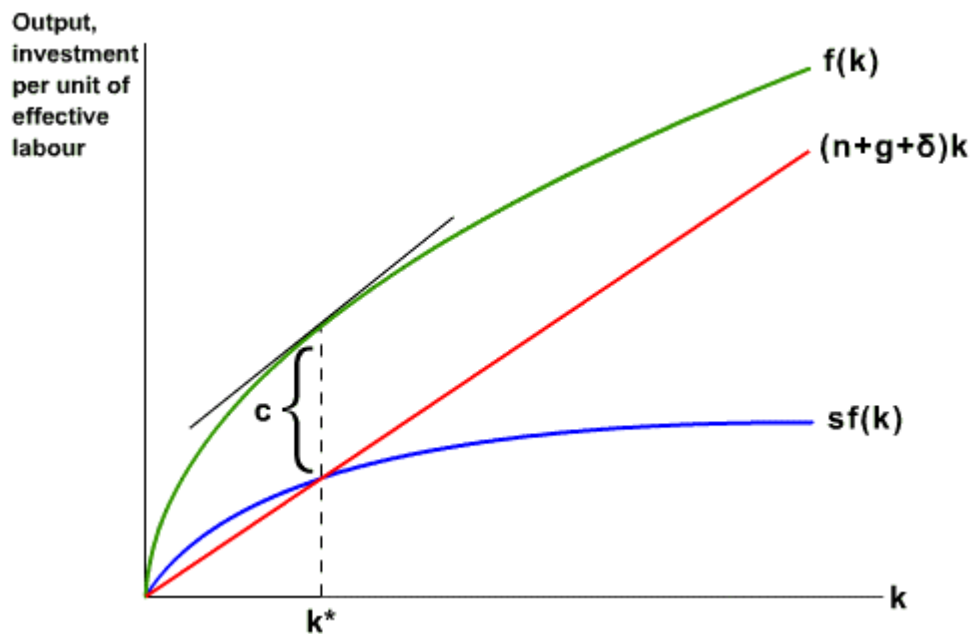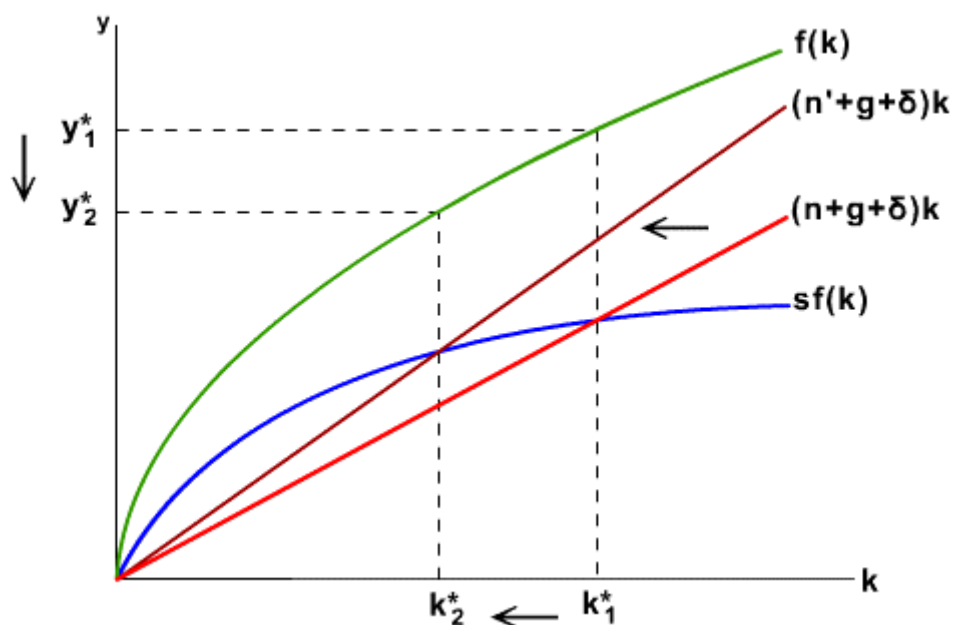
2.5   Population growth and the steady state

- The higher the rate of population growth, the lower the steady state **k**.

- Since Solow assumed that the production function is subject to constant return to scale, technically then total output **Y** increases at the same rate as all inputs. But because economic growth is defined as the growth of per capital output, the steady state with population growth still does not imply permanent economic growth. That

is, the growth rate of **Y**, **K** and **L** equals to **n**, but the growth rate of **y** an **k** remains zero.

- The follow model does not explain the rate of population growth. It merely assumes some rate of population growth. But, it highlights the role that population growth plays in determining an economy's steady state level of per capita output.
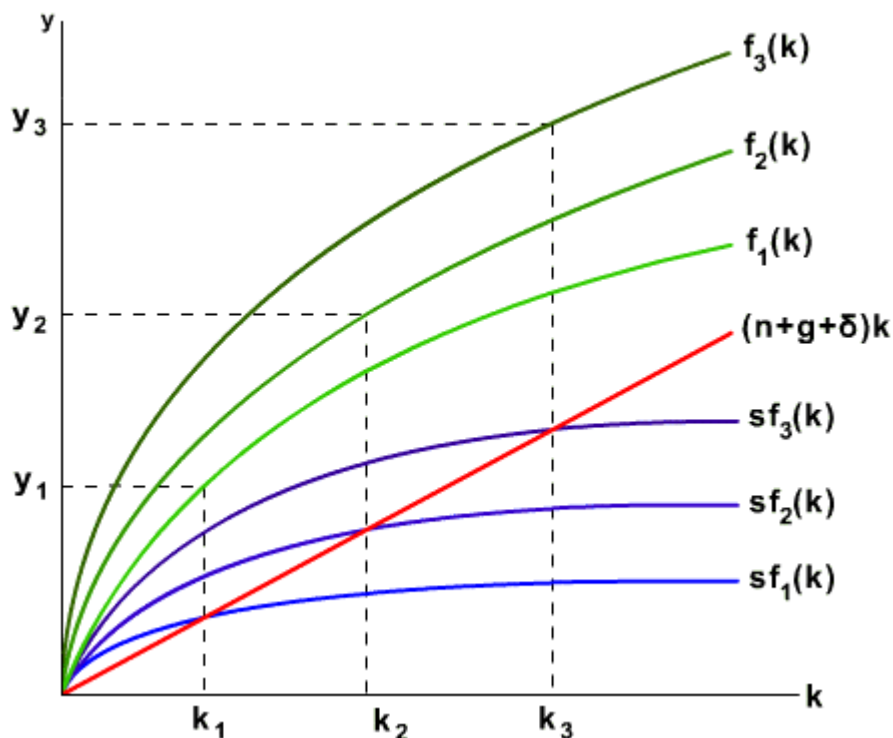

- The Solow model predicts that, ceteris paribus, countries with higher rate of population growth will have lower levels of per capital output.

2.6 Technological progress and the steady state

- As technological progress occurs, the entire production function shifts up. This also raises the saving function and thus the steady - state level of **k** also rises.

2.7 Implications around the long-run equilibrium

2.7.1 The effect on output in long run

- The long-run effect of a rise in saving on output:

$$y^* = f(k^*) \implies \frac{\partial y^*}{\partial s} = f'(k^*) \frac{\partial k^*(s,n,g,\delta)}{\partial s} \quad \text{------------------------------(20)}$$

- Next, find $\dfrac{\partial k^*}{\partial s}$:

With $\dot{k} = 0$: $sf\big(k^*(s,n,g,\delta)\big) = (n+g+\delta)k^*(s,n,g,\delta)$

Take the derivative of both sides with respect to **s**:

$$sf'(k^*)\frac{\partial k^*}{\partial s} + f(k^*) = (n+g+\delta)\frac{\partial k^*}{\partial s} \quad \text{------------------------------(21)}$$

$$f(k^*) = (n+g+\delta)\frac{\partial k^*}{\partial s} - sf'(k^*)\frac{\partial k^*}{\partial s}$$

$$= \frac{\partial k^*}{\partial s}\left[(n+g+\delta) - sf'(k^*)\right]$$

$$\frac{\partial k^*}{\partial s} = \frac{f(k^*)}{(n+g+\delta) - sf'(k^*)} \quad\text{-- (22)}$$

Subs. **(22)** into **(20)** :

$$\frac{\partial y^*}{\partial s} = \frac{f'(k^*)\,f(k^*)}{(n+g+\delta) - sf'(k^*)} \quad\text{------------------------------- (23)}$$

- Convert **(23)** to an elasticity by multiplying both side by $\dfrac{s}{y^*}$:

$$\frac{s}{y^*}\frac{\partial y^*}{\partial s} = \frac{s}{f(k^*)}\frac{f'(k^*)\,f(k^*)}{(n+g+\delta) - sf'(k^*)}$$

Using the fact that $sf'(k) = (n+g+\delta)k^*$ to substitute for **s** :

$$\frac{s}{y^*}\frac{\partial y^*}{\partial s} = \frac{(n+g+\delta)k^*\,f'(k^*)}{f(k^*)\left[\dfrac{(n+g+\delta) - (n+g+\delta)k^*\,f'(k^*)}{f(k^*)}\right]}$$

$$= \frac{\dfrac{k^*\,f(k^*)(n+g+\delta)\,f'(k^*)}{f(k^*)}}{f(k^*)(n+g+\delta)\left[1 - \dfrac{k^*\,f'(k^*)}{f(k^*)}\right]}$$

$$= \frac{\dfrac{k^*\,f'(k^*)}{f(k^*)}}{1 - \left[\dfrac{k^*\,f'(k^*)}{f(k^*)}\right]}$$

- If markets are competitive, capital earns its marginal product, the share of total

income that goes to capital on the balance growth path is α_k **(k*)**.

- In most counters, the share of income paid to capital α_k **(k*)** is about one- third,

  thus $\dfrac{s}{y^*}\ \dfrac{\partial\, y^*}{\partial\, s} = \dfrac{0.3333}{0.6667} = 0.5$

  $\Rightarrow$ the elasticity of output with respect to the saving rate in the long run is about one-half.

- For a 10% increase in the saving rate from, say, 20% of output to 22%, raises output per worker in long run by about 5%. Even a 50% increase in the saving rate raises output per worker in the long run by about 22%.

- Significant changes in saving have only moderate effects on the level of output on the balanced growth path.

2.7.2 Convergence and the Solow model

A) Absolute convergence

- It assumer that poor economies tend to grow faster per capita than rich economies without conditioning on any other characteristics of economies.

- Consider a group of closed economies that are structurally similar in the sense that they have the same values of **s**, **n**, **g** and δ, and also have the same production function.

- Assume that the only difference among the economies is the initial quantity of capital per person.

- Regions or countries with lower starting values of **k(0)** have higher per capital growth rates and tend to catch up or converge to these with higher capital - labor ratios. Thus all the economies have the same steady-state values **k*** and **y***.

  Divide both sides of **(16)** by **k**: $\dfrac{\dot{k}(t)}{k} = \dfrac{s\,f\big(k(t)\big)}{k} - (n+g+\delta)$ ------------- **(25)**

- $\dfrac{sf(k)}{k}$ is a downward- sloping curve which asymptotes to infinity at **k = 0** and approaches zero as **k** tends to infinity. $\mathbf{n+g+\delta}$ is a horizontal line.

- The vertical distance between the $\dfrac{sf(k)}{k}$ curve and the $\mathbf{n+g+\delta}$ line equals the growth rate of capital per person.

- Since the $\dfrac{sf(k)}{k}$ curve and the $\mathbf{n+g+\delta}$ line intersect once only, the steady - state capital - labor ratio **k* > 0** exists and is unique.
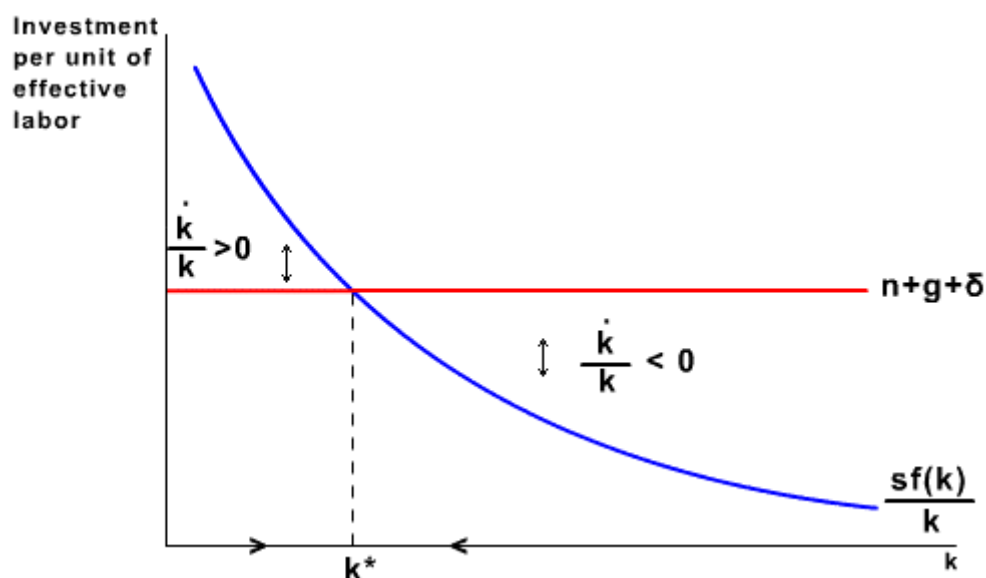

- When **k** is relatively low, the average product of capital, $\dfrac{f(k)}{k}$, is relatively high. The gross investment per unit of capital, $\dfrac{sf(k)}{k}$, is also high. Consequently, the growth rate, $\dfrac{\dot{k}}{k}$ is also high.

- If the economy starts with **k(0) > k***, then the growth rate of **k** is negative, and **k** falls over time. The growth rate increases and approaches zero as **k** approaches **k***.

- **(25)** implies that there exists a negative relationship between the growth rate of capital per person and capital per person:

$$\frac{\partial \dfrac{\dot{k}(t)}{k}}{\partial k} = \frac{s\left[k\,f\,'(k) - f(k)\,(1)\right]}{k^2} = \frac{s\left[f\,'(k) - \dfrac{f(k)}{k}\right]}{k} < 0 \quad\text{.................(26)}$$

B) Conditional convergence

- The Solow model predicts conditional convergence. Each economy converge to its own steady state and that the speed of this convergence relates inversely to the distance from the steady state.

- It is true empirically that countries with higher level of real per capital GDP tend to have higher saving rate, i.e. $s_{poor} < s_{rich}$ and hence $k^*_{poor} < k^*_{rich}$.

- The rich economy would be predicted to grow faster per capita than the poor economy. A poor country may grow at a slower rate than a rich one.
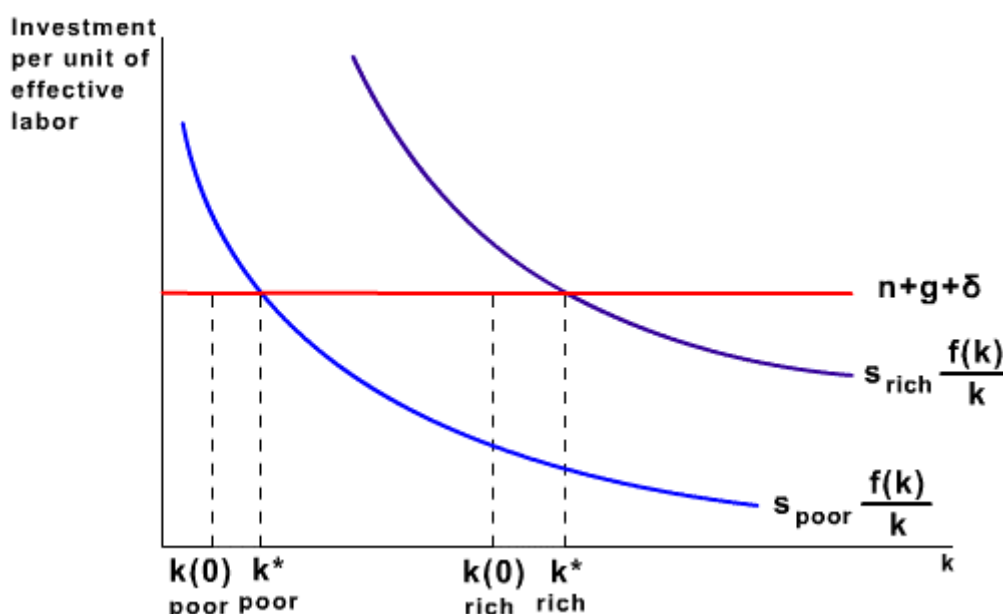
C) Case study: The Asian "Economic Miracle"

- The economies of Hong Kong, Korea, Singapore and Taiwan stand out because of their spectacular rates of economic growth over the past four decades.

- Hong Kong and Singapore currently enjoy real per capital incomes of over $20,000 , placing them ahead of mort European countries. Korea increased its per capital income more than co-fold from less than $1000 in 1950 to over $10,000 by 1996.

Taiwan raised its per capital income form $900 in 1950 to over $13,000 in the mid-1990s.

- Paul Krugman applies the logic of the Solow model to show that the rapid East Asian economic growth must slow because diminishing returns will set in.

- Aylwih Young found that the rate of technological progress was near zero in Singapore and that for the other Asian Tigers it was not much greater than technological progress in the most developed economies.

- Krugman explained that the East Asian economies grew because they sacrificed the percent for the future and increased their stocks of labor, capital, and human capital. But the labor force participation rater cannot increase much more, hence diminishing returns to capital and human capital will set in. The rapid growth rates of the Asian Tigers are not a miracle, and they will not continue indefinitely.

- Krugman used the Solow growth model to predict that long- run growth rates will be equal to the rate of technological progress. Without technological progress, increase in factors such as capital and human capital can have only medium - term effects as the economy transitions to a higher steady state but diminishing returns prevent any permanent increase in the rate of growth.

- In fast, total factor productivity growth in the Asian Tigers has not been noticeably faster than that in the so- called G-7 countries during 1960s to 1990s. Aylwih young said that the Asian Tigers' rate of technological progress had been nothing exceptional.

- To conclude, Asia's long-run growth rate would depend on future technological progress.

D) Technological progress and convergence in the long run

- The rate of technological progress determines the rate at which the economy's long - run growth path (LGP) slopes upward, and the variables such as **s**, δ, **n** determine the level of the long-run growth path.
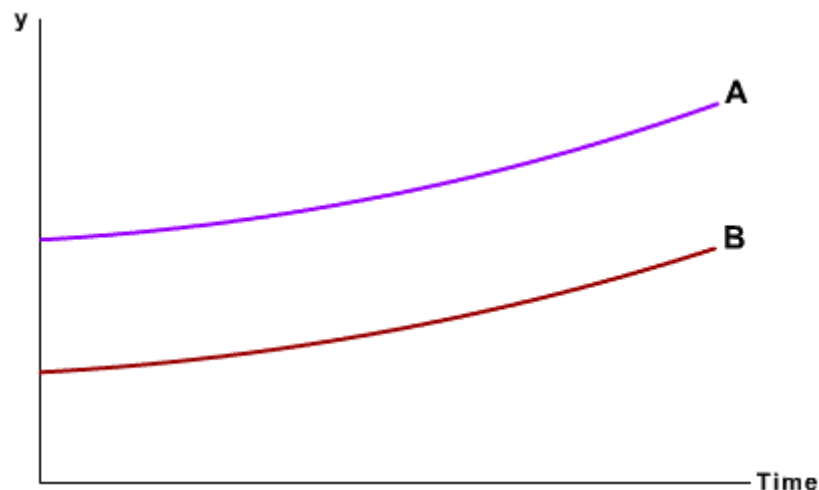
- Country A has a higher raving rate or a lower rate of population growth that country B. Thus, with identical rates of technological progress and identical rate of economic growth, these is neither convergence nor divergence.
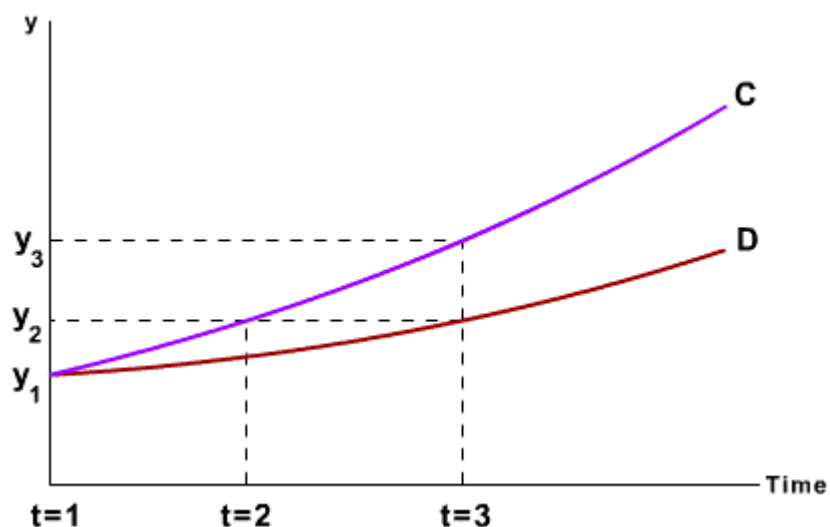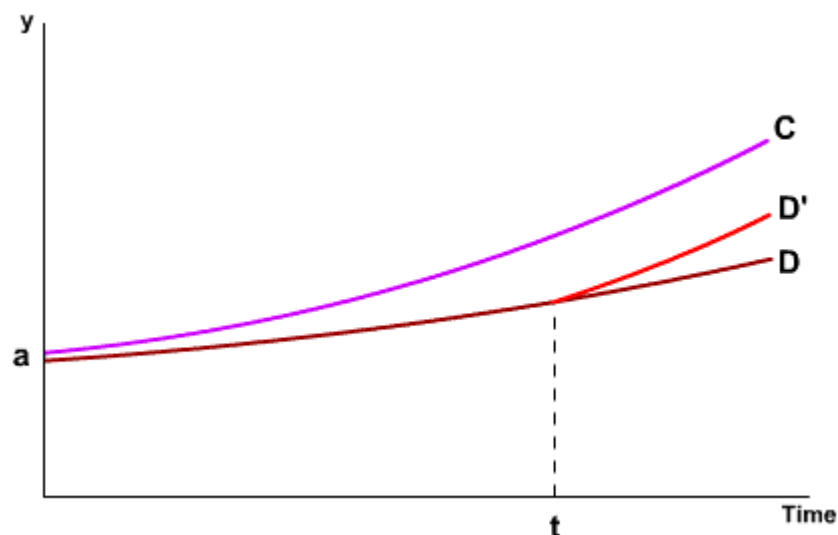

- Assume that technological progress is faster in country C than in country D. Despite having same saving rates, depreciation rates, and population growth rates, the two economies diverge.

- If a low - income country B achieves the same saving, depreciation, population growth, and technological progress rate as high - income country A, it will converge to country A's standard of living. This is the conditional convergence predicted by the Solow model. Note that the slope of country B 's dashed transition path just after **t** is steep than A 's growth path.
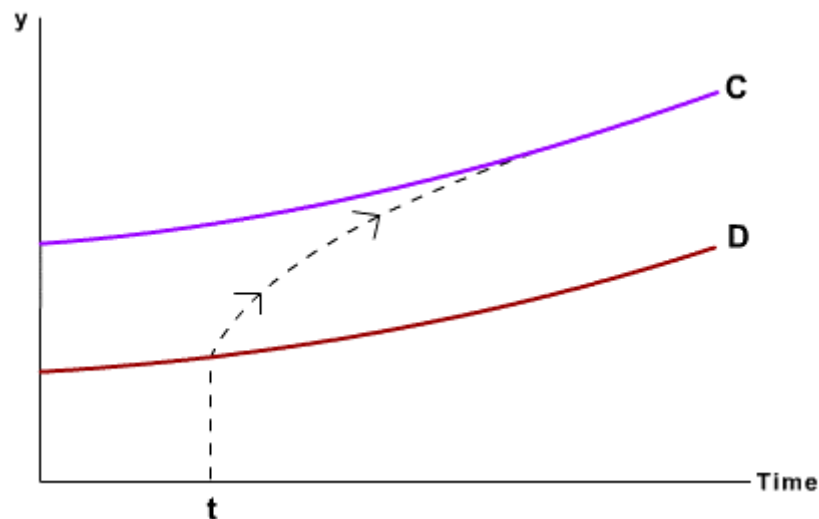
- Krugman suggested that if the Asian Tigers have higher steady states than do developed countries such as the United States, their LGPS lie above that of the United States and the Asian Tigers will eventually have the highest per capita output levels in the world. But the height of the LGP depends not only on rates of saving, population growth, or depreciation but also on the accumulated level of technology.

E) The level of technology and the growth path

- Suppose that country D manages to increase its rate of technological progress to the same rate enjoyed by country C by either opening its borders to more trade and investment or adopting policies to stimulate more innovation and invention.

- Even with the same rate of technological progress, because country D stated out at a lower lever of technology at time **t**, because of slower technological progress in the part, country D will diverge from country C forever. Country D will never enjoy the same level of per capita output as country C.

- Assume that country D starts at a lower steady state, point a, and begins with a lower level of accumulated technology. Country D could accomplish faster growth by raising its rate of technological progress above that of country C.
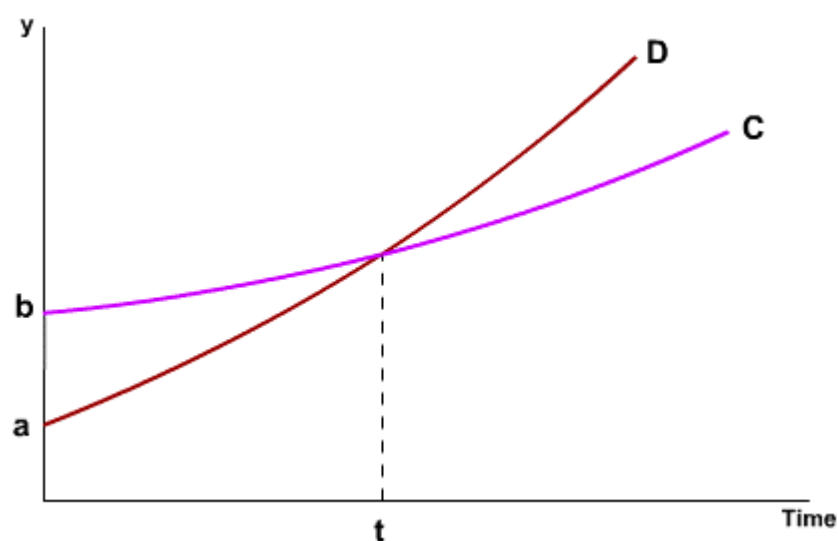
- Country D could also catch up to C by implementing policies to raise stead - state variables such as **s**, δ and **n** above country C's levels in order to complete convergence.

- Complete convergence requires that both countries have the same steady states and the same long - run rates of technological progress.

- Whether the transition to a higher steady state will lead to complete convergence is

not clear since there are limits to how far the saving, depreciation, and population growth rates can be altered

F) Long- run convergence requires faster technological progress

- Assume that the developing economy's production function $f_1(k)$ is only one- half as high as the production function $f_2(k)$ of a developed economy. The rate of saving is 40% in the developing economy and 20% in the developed economy. Since the production function of the developed economy is twice as high as the production function of the developing economy, the saving functions of both countries are the same. i.e. $0.4f_1(k) = 0.2f_2(k)$.

- If the depreciation rate and population growth are identical in both countries, then the steady state per capita income is twice as high $100 vs. $50 in the developed country.

- If the developing country raises its saving rate to 50%. Per capita income will only rise to $80 in the medium run.

- Because of the developing economy's lower production function, the income gap will not be closed. Perhaps a saving rate of about 70% can complete the gap, an unrealistically – high rate of saving.

- Such a high rate of saving leaves a smaller level of consumption and hence, low- income economies cannot close the income gap on the basis of factor accumulation only.

- Equal rate of technological progress would only cause both countries production functions to shift up at the same rates, but the proportional income gap would remain unchanged. Only if the production function rises more rapidly via faster speed of technological progress, can identical steady state level of income and welfare be achieved.
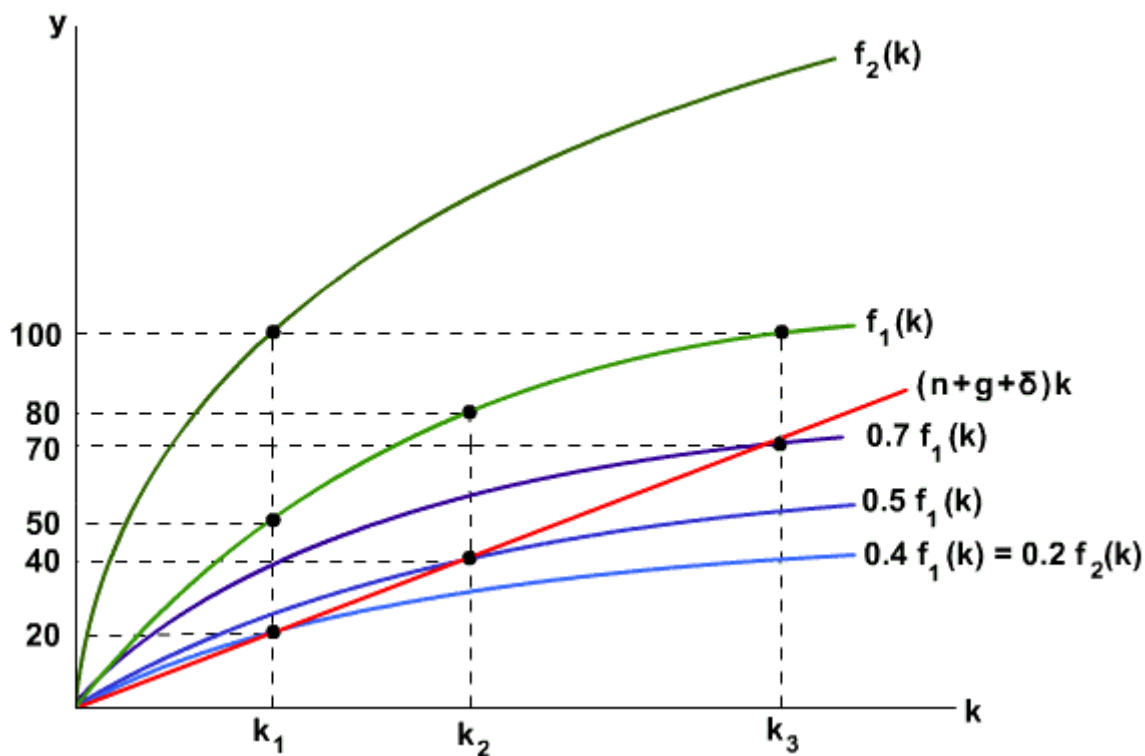
2.8 Sources of variation in output per worker in the Solow model

- The Solow model identities two possible sources of variation in output per work: differences in capital per worker (**K/L**) and differences in the effectiveness of labor (**A**).

A) Capital per worker

- In fact, variations in the accumulation of physical capital do not account for a significant part of either worldwide economic growth or cross - country income differences.

    Explanation:

(1) The required differences in capital are too large when trying to account for large differences in income on the basis of differences in capital.

- Assume that output per worker in the United States today is ten times larger than it was a hundred years ago.

- Let capital share $\alpha_k = \dfrac{1}{3}$, a tenfold difference in output per worker requires a difference of a factor of $10^{\frac{1}{\alpha}} = 1000$.

- Indeed, the U.S. capital stock per worker is roughly ten times larger than it was a hundred years ago, note thousand times larger.

- Kaldor said that in most of the major industrialized countries over the part century, the growth rates of labor, capital and output are each roughly constant. The growth rates of output and capital are larger than the growth rate of labor so that output per worker and capital per worker are rising. Although capital - output ratios vary across countries, the variation is not great.

- In sum, differences in capital per worker are smaller than those needed to account for the differences in output per worker.

(2) Attributing differences in output to differences in capital without differences in the effectiveness of labor implies large variation in the rate of return on capital.

- $\mathbf{MP_k} : \mathbf{f(k) = k^\alpha \Rightarrow f'(k) = \alpha\, k^{\alpha - 1} = \alpha \left(\dfrac{k^\alpha}{k} \right) = \alpha\, y^{\frac{\alpha - 1}{\alpha}}}$ ....(27)

where $\mathbf{y^{\frac{\alpha - 1}{\alpha}} = y^{1 - \frac{1}{\alpha}} = y \big/ y^{\frac{1}{\alpha}}}$ provided the numerator $\mathbf{y = k^\alpha}$ and the denominator $\mathbf{y^{\frac{1}{\alpha}} = k}$.

Where α is the elasticity of output with respect to capital.

- **(27)** implies that the elasticity of the marginal product of capital with respect to output is $-\dfrac{1 - \alpha}{\alpha}$.

- Let $\alpha = \dfrac{1}{3}$, a tenfold difference in output per worker arising from differences in capital per worker implies a hundredfold difference in the $\mathbf{MP_K}$ i.e.

$$y^{\frac{\alpha-1}{\alpha}} = 10^{1-\frac{1}{3}/\frac{1}{3}} = 100$$

- There is no evidence of such larger the difference in rate to return. If rates of return were larger by a factor of ten or hundred in poor countries than in rich counties, there would be many incentives to invest in poor countries. Empirically, there are not the facts.

B) Effectiveness of labor

- Attributing differences in standards of living to differences in the effectiveness of labor does not require large differences in capital or in rates of return.

- Along a balanced growth path, capital is growing at the same rate as output and the MP_K is constant.

- The Solow model identities the behavior of the effectiveness of labor as the driving force of growth. However, the growth of the effectiveness of labor is regarded as exogenous.

- The Solow model does not identify what the "effectiveness of labor" is. It is just a factor other than labor and capital that affects output.

- Interpretations of the effectiveness of labor are : knowledge, the education and skills of labor force the strength of property rights, the quality of infrastructure, cultural attitudes toward entrepreneurship and work.

2.9 The nature of knowledge

- Basic scientific knowledge with broad applicability such as the theory of quantum mechanics.

- Invention of applied technologies useful in broad classes of goods such as how to start a particular lawn mower on a cold morning.

- Invention of new products.

- Improvements in the design and use of products after their invention.

- Different types of knowledge all play important rules in economic growth.

- Factors determining the allocation of resources to R &D:

a. Support for basic scientific research

   It is supported by governments, charities, and wealthy individuals and is pursued by individuals motivated by this support, by desire for fame, and by low of knowledge.

b. Private incentives for R&D and innovation

- Consumer – surplus effect   is that the individuals or firms licensing ideas from innovators obtain some surplus. This is a positive externality from R&D.

- Business – stealing effect   is that the introduction of a superior technology makes existing technologies less attractive, and thus harms the owners of those existing technologies. This externality is negative.

- R&D effect   is that innovators are generally assumed not to control the use of their knowledge in the production of additional knowledge until a patent is granted. This externality is positive.

- However, the net effect of these three externalities is unclear.

c. Alternative opportunities for talented individuals

- Baumol (1990) and Murphy, Shleifer, and Vishny (1991) observe that major innovations and advances in knowledge are often the result of the work of extremely talented individuals. There individuals have choices other than just pursuing innovations and producing goods

- Three factors affecting talented individuals' decisions whether to pursue activities that are socially productive:

1) The size of the market

   The larger is the market from which a talented individual can reap returns, the greater are the incentives to enter a given activity.

2) The degree of diminishing returns

   Allocating more time to create inventions may not offer more inventions.

3) The ability to keep the returns from one's activities

   Clear property right may tend to encourage entrepreneurship.